

# Fayette Academy

## Dual Enrollment Calculus

### Summer Packet

#### Directions:

1. This is due Friday, August 5, 2022
2. For all work, write Very clearly and legibly.
3. Copy each problem given onto your own notebook.
4. Show each and every step in obtaining your solution.
5. Circle your answer(s) and if asked, clarify your reasoning.
6. Write all answers, including your graphs, on the answer sheet.

#### Notes:

To be prepared for your first examination, these are the core concepts you need to have mastered to be potentially successful for Dual Enrollment Calculus, a very challenging course.

#### Finding Intercepts

In exercises 1 – 4, find any intercepts.

1.  $y = 5x - 8$

2.  $f(x) = x^2 - 8x + 12$

3.  $= \frac{x-3}{x-4}$

4.  $f(x) = (x-3)\sqrt{x+4}$

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#### Testing for Symmetry

In exercises 5 – 8, test for symmetry with respect to each axis and to the origin.

5.  $y = x^2 + 4x$

6.  $y = x^4 - x^2 + 3$

7.  $y^2 = x^2 - 5$

8.  $xy = -2$

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#### Using Intercepts and Symmetry to Sketch a Graph

In exercises 9 – 14, find any intercepts and test for symmetry. Then sketch the graph of the equation.

9.  $y = -\frac{1}{2}x + 3$

10.  $y = -x^2 + 4$

11.  $y = 9x - x^3$

12.  $y^2 = 9 - x$

$$13. y = 2\sqrt{4-x}$$

$$14. y = |x-4| - 4$$

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### **Finding Points of Intersection**

In exercises 15 – 18, find the points of intersection of the graphs of the equations.

$$15. \begin{cases} 5x+3y = -1 \\ x-y = -5 \end{cases}$$

$$16. \begin{cases} 2x+4y=9 \\ 6x-4y=7 \end{cases}$$

$$17. \begin{cases} x-y=-5 \\ x^2-y=1 \end{cases}$$

$$18. \begin{cases} x^2+y^2=1 \\ -x+y=1 \end{cases}$$

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### **Finding the Slope of a Line**

In exercises 19 – 20, plot the pair of points and find the slope of the line passing through them.

$$19. \left(\frac{3}{2}, 1\right), \left(5, \frac{5}{2}\right)$$

$$20. (-7, 8), (-1, 8)$$

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### **Finding an Equation of a Line**

In exercises 21 – 24, find an equation of the line that passes through the point and has the sketch the line.

indicated slope. Then

$$21. (3, -5); m = \frac{7}{4}$$

$$22. (-8, 1); m \text{ is undefined}$$

$$23. (-3, 0); m = -\frac{2}{3}$$

$$24. (5, 4); m = 0$$

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### **Finding the slope and y-intercept**

In exercises 25 – 26, find the slope and the y-intercept (if possible) of the line.

$$25. y-3x = 5x$$

$$26. 9-y = x$$

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### **Sketching a Line in the Plane**

In exercises 27 – 30, sketch the graph of the equation.

$$27. y=6$$

$$28. x=-3$$

$$29. y=4x-2$$

$$30. 3x+2y=12$$

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### **Finding an Equation of a Line**

In exercises 31 – 32, find an equation of the line that passes through the points. Then sketch the line.

31.  $(0, 0), (8, 2)$

32.  $(-5, 5), (10, -1)$

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**Finding Equations of Lines**

Find equations of the lines passing through  $(-3, 5)$  and having the following characteristics.

33. (a) Slope of  $\frac{7}{16}$  (b) Parallel to the line  $5x - 3y = 3$  (c) Perpendicular to the line  $3x + 4y = 8$   
(d) Parallel to the  $y$ -axis

Find equations of the lines passing through  $(2, 4)$  and having the following characteristics.

34. (a) Slope of  $-\frac{2}{3}$  (b) Perpendicular to the line  $x + y = 0$  (c) Parallel to the line  $3x - y = 0$   
(d) Parallel to the  $x$ -axis

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**Rates of Change**

35. The purchase price of a new machine is \$12,500, and its value will decrease by \$850 per year. Use this information to write a linear equation that gives the value  $V$  of the machine  $t$  years after it is purchased. Find its value at the end of 3 years.

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**Break-Even Analysis**

36. A contractor purchases a piece of equipment for \$36,500 that costs an average of \$9.25 per hour for fuel and maintenance. The equipment operator is paid \$13.50 per hour, and customers are charged \$30 per hour.

- (a) Write a linear equation for the cost  $C$  of operating this equipment for  $t$  hours.  
(b) Write a linear equation for the revenue  $R$  derived from  $t$  hours of use.  
(c) Find the break-even point for this equipment by finding the time at which  $R = C$ .

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**Evaluating a Function**

In exercises 37 – 40, evaluate the function at the given value(s) of the independent variable. Simplify the results.

37.  $f(x) = 5x + 4$  (a)  $f(0)$  (b)  $f(5)$  (c)  $f(-3)$  (d)  $f(t + 1)$

38.  $f(x) = x^3 - 2x$  (a)  $f(-3)$  (b)  $f(2)$  (c)  $f(-1)$  (d)  $f(c - 1)$

39.  $f(x) = 4x^2$ ; Find  $\frac{f(x + \Delta x) - f(x)}{\Delta x}$ .

40.  $\frac{f(x) - f(1)}{x - 1}$

### Finding the Domain and the Range of a Function

In exercises 41 – 44, find the domain and range of each function.

41.  $f(x) = x^2 + 3$

42.  $g(x) = \sqrt{6 - x}$

43.  $f(x) = -|x + 1|$

44.  $h(x) = \frac{2}{x + 1}$

### Sketching a Graph of a Function

In exercises 45 – 46, sketch a graph of the function and find its domain and range. Use a graphing utility to verify your graph

45.  $f(x) = \frac{4}{2x - 1}$

46.  $g(x) = \sqrt{x + 1}$

### Using the Vertical Line Test

In exercises 47 – 48, use the Vertical Line Test to determine whether  $y$  is a function of  $x$ .

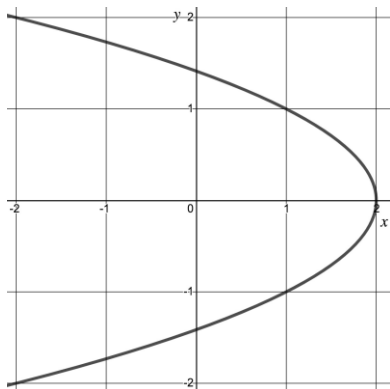
47.  $x + y^2 = 2$

48.  $x^2 - y = 0$

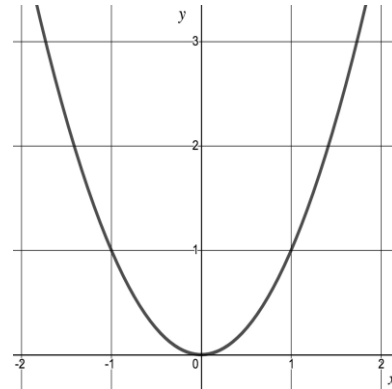
### Deciding Whether an Equation is a Function

In exercises 49 – 50, determine whether  $y$  is a function of  $x$ .

49.  $xy + x^3 - 2y = 0$

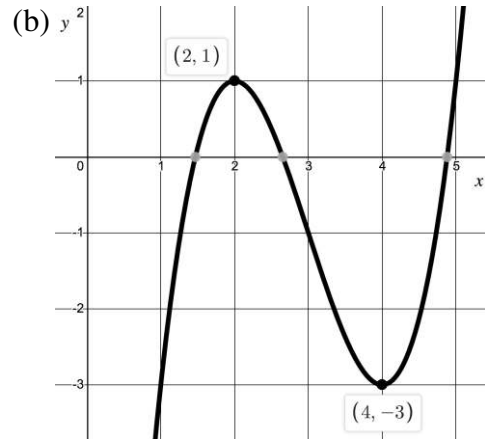
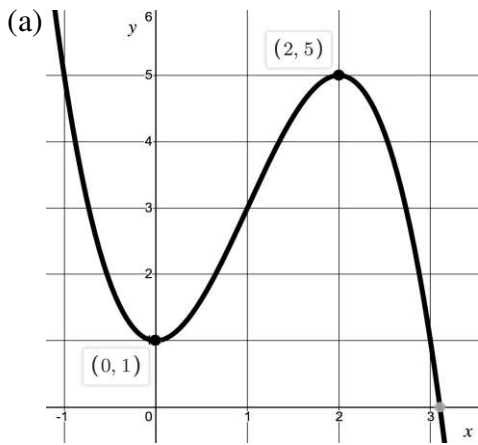


50.  $x = 9 - y^2$



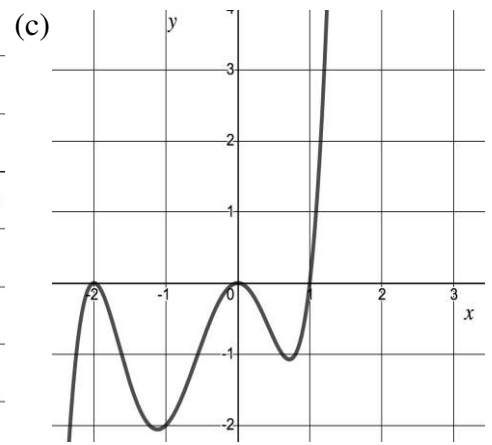
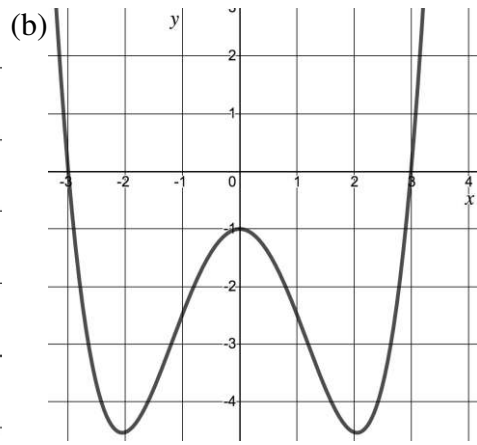
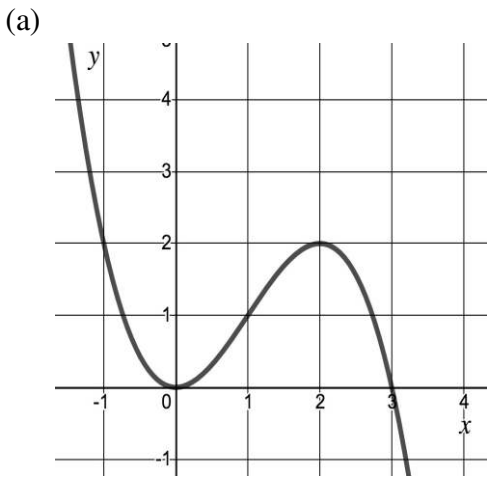
### Transformations of Functions

51. Use a graphing utility to graph  $f(x) = x^3 - 3x^2$ . Use the graph to write a formula for the function  $g$  shown in the figure.



### Think About It

52. What is the minimum degree of the polynomial function whose graph approximates the given graph? What sign must the leading coefficient have?



### Finding Composite Functions

In exercises 53 – 54, find the composite functions  $f \circ g$  and  $g \circ f$ . Find the domain of each composite function. Are the two composite functions equal?

53.  $f(x) = 3x + 1$   
 $g(x) = -x$

54.  $f(x) = \sqrt{x - 2}$   
 $g(x) = x^2$

### **Even and Odd Functions and Zeros of Functions**

In exercises 55 – 56, determine whether the function is even, odd, or neither. Then find the zeros of the function. Use a graphing utility to verify your result.

55.  $f(x) = x^4 - x^2$

56.  $f(x) = \sqrt{x^3 + 1}$

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### **Degrees to Radians**

In exercises 57 – 60, convert the degree measure to radian measure as a multiple of  $\pi$  and as a decimal accurate to three places.

57.  $340^\circ$

58.  $300^\circ$

59.  $-480^\circ$

60.  $-900^\circ$

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### **Radians to Degrees**

In exercises 61 – 64, convert the radian measure to degree measure.

61.  $\frac{\pi}{6}$

62.  $\frac{11\pi}{4}$

63.  $-\frac{2\pi}{3}$

64.  $-\frac{13\pi}{6}$

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### **Evaluating Trigonometric Functions**

In exercises 65 – 70, evaluate the sine, cosine, and tangent of the angle. Do not use a calculator.

65.  $-45^\circ$

66.  $240^\circ$

67.  $\frac{13\pi}{6}$

68.  $-\frac{4\pi}{3}$

69.  $405^\circ$

70.  $180^\circ$

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### **Evaluating Trigonometric Functions Using Technology**

In exercises 71 – 76, use a calculator to evaluate the trigonometric function. Round to four decimal places.

71.  $\tan 33^\circ$

72.  $\cot 401^\circ$

73.  $\sec 125^\circ$

74.  $\csc 29^\circ$

75.  $\sin\left(-\frac{\pi}{9}\right)$

76.  $\cos\left(-\frac{3\pi}{7}\right)$

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### **Solving a Trigonometric Equation**

In exercises 77 – 82, solve the equation for  $\theta$ , where  $0 \leq \theta < 2\pi$ .

77.  $2\cos\theta + 1 = 0$

78.  $2\cos^2\theta = 1$

79.  $2\sin^2\theta + 3\sin\theta + 1 = 0$

80.  $\cos^3\theta = \cos\theta$

81.  $\sec^2\theta - \sec\theta - 2 = 0$

82.  $2\sec^2\theta + \tan^2\theta - 5 = 0$

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### Sketching the Graph of a Trigonometric Function

In exercises 83 – 90, sketch the graph of the function.

83.  $y = 9\cos x$                       84.  $y = \sin \pi x$                       8t.  $y = 3\sin \frac{2x}{5}$                       86.  $y = 8\cos \frac{x}{4}$

87.  $y = \frac{1}{3}\tan x$                       88.  $y = \cot \frac{x}{2}$                       89.  $y = -\sec 2\pi x$                       90.  $y = -4\csc 3x$

### Finding Tangent Lines

91. Consider the circle  $x^2 + y^2 - 6x - 8y = 0$ .

- (a) Find the center and radius of the circle.
- (b) Find an equation of the tangent line to the circle at the point  $(0, 0)$ .
- (c) Find an equation of the tangent line to the circle at the point  $(6, 0)$ .
- (d) Where do the two tangent lines intersect?

92. There are two tangent lines from the point  $(0, 1)$  to the circle  $x^2 + (y+1)^2 = 1$ . Find equations of the two tangent lines by using the fact that each tangent line intersects the circle at exactly one point.

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### Heaviside Function

93. The Heaviside function  $H(x)$  is widely used in engineering applications.  $H(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$

Sketch the graph of the Heaviside function and the graphs of the following by hand.

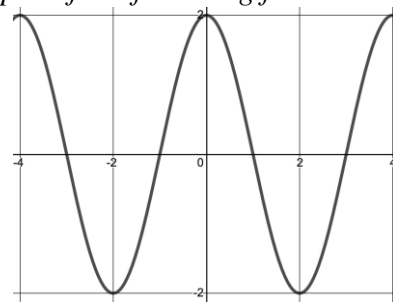
- (a)  $H(x) - 2$                       (b)  $H(x-2)$                       (c)  $-H(x)$
- (d)  $H(-x)$                       (e)  $\frac{1}{2}H(x)$                       (f)  $-H(x-2) + 2$

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### Sketching Transformations

94. Consider the graph of the function  $f$  shown. Use this graph to sketch the graphs of the following functions.

- (a)  $f(x+1)$                       (b)  $f(x)+1$                       (c)  $2f(x)$
- (d)  $f(-x)$                       (e)  $-f(x)$                       (f)  $|f(x)|$
- (g)  $f(|x|)$



## Maximum Area

95. A rancher plans to fence a rectangular pasture adjacent to a river. The rancher has 100 meters of fencing, and no fencing is needed along the river.

- (a) Write the area  $A$  of the pasture as a function of  $x$ , the length of the side parallel to the river. What is the domain of  $A$ ?
- (b) Graph the area function and estimate the dimensions that yield the maximum amount of area for the pasture. (c) Find the dimensions that yield the maximum amount of area for the pasture by completing the square.

96. A rancher has 300 feet of fencing to enclose two adjacent pastures.

- (a) Write the total area  $A$  of the two pastures as a function of  $x$ . What is the domain of  $A$ ?
- (b) Graph the area function and estimate the dimensions that yield the maximum amount of area for the pastures. (c) Find the dimensions that yield the maximum amount of area for the pasture by completing the square.
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## Graphing an Equation

97. Explain how you would graph the equation  $y + |y| = x + |x|$ .

Then sketch the graph.

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## Composite Functions

98. Let  $f(x) = \frac{1}{1-x}$

- (a) What are the domain and range of  $f$ ?
- (b) Find the composition  $f(f(x))$ . What is the domain of this function?
- (c) Find  $f(f(f(x)))$ . What is the domain of this function?
- (d) Graph  $f(f(f(x)))$ . Is the graph a line? Why or why not?
- 

## Slope of a Tangent Line

99. One of the fundamental themes of calculus is to find the slope of the tangent line to a curve at a point. To see how this can be done, consider the point  $(2, 4)$  on the graph of  $f(x) = x^2$ .

- (a) Find the slope of the line joining  $(2, 4)$  and  $(3, 9)$ . Is the slope of the tangent line at  $(2, 4)$  greater than or less than this number?
- (b) Find the slope of the line joining  $(2, 4)$  and  $(1, 1)$ . Is the slope of the tangent line at  $(2, 4)$  greater than or less than this number?
- (c) Find the slope of the line joining  $(2, 4)$  and  $(2.1, 4.41)$ . Is the slope of the tangent line at  $(2, 4)$  greater than or less than this number?



(d) Find the slope of the line joining  $(2, 4)$  and  $(2+h, f(2+h))$  in terms of the nonzero number  $h$ . Verify that  $h=1, -1$ , and  $0.1$  yield the solutions to parts (a) – (c) above.

(e) What is the slope of the tangent line at  $(2, 4)$ ? Explain how you arrived at your answer.

**100.** Again, one of *THE* fundamental themes of calculus is to find the slope of a tangent line to a curve at a point.

To see how this can be done one more time, consider the function  $f(x) = x$  and the point  $(4, 2)$ .

(a) Find the slope of the line joining  $(4, 2)$  and  $(9, 3)$ . Is the slope of the tangent line at  $(4, 2)$  greater than or less than this number?

(b) Find the slope of the line joining  $(4, 2)$  and  $(1, 1)$ . Is the slope of the tangent line at  $(4, 2)$  greater than or less than this number?

(c) Find the slope of the line joining  $(2, 4)$  and  $(4.41, 2.1)$ . Is the slope of the tangent line at  $(4, 2)$  greater than or less than this number?

(d) Find the slope of the line joining  $(4, 2)$  and  $(4+h, f(4+h))$  in terms of the nonzero number  $h$ .

(e) What is the slope of the tangent line at  $(4, 2)$ ? Explain how you arrived at your answer.

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**Fayette Academy  
Dual Enrollment Calculus  
Summer Packet**

**Name:** \_\_\_\_\_

**Signature:** \_\_\_\_\_

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2. _____	22. _____	36a. _____
3. _____	23. _____	36b. _____

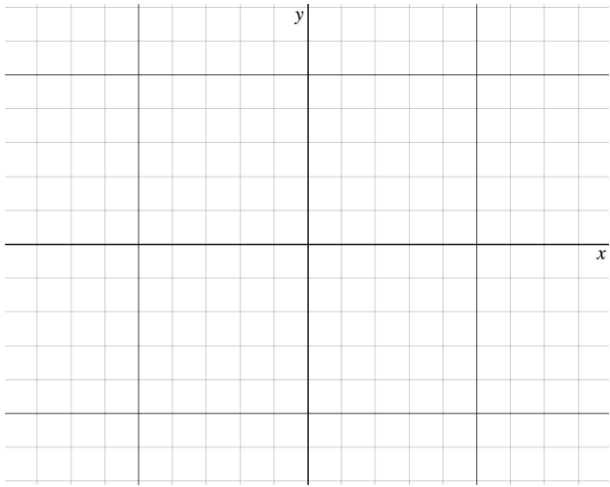
4. _____	24. _____	36c. _____
5. _____	25. _____	37a. _____
6. _____	26. _____	37b. _____
7. _____	27. -----(See Attached Graph)-----	37c. _____
8. _____	28. -----(See Attached Graph)-----	37d. _____
9. _____	29. -----(See Attached Graph)-----	38a. _____
10. _____	30. -----(See Attached Graph)-----	38b. _____
11. _____	31. _____	38c. _____
12. _____	32. _____	38d. _____
13. _____	33a. _____	39. _____
14. _____	33b. _____	40. _____
15. _____	33c. _____	41. _____
16. _____	33d. _____	42. _____
17. _____	34a. _____	43. _____
18. _____	34b. _____	44. _____
19. _____	34c. _____	45. _____

20. _____	34d. _____	46. _____
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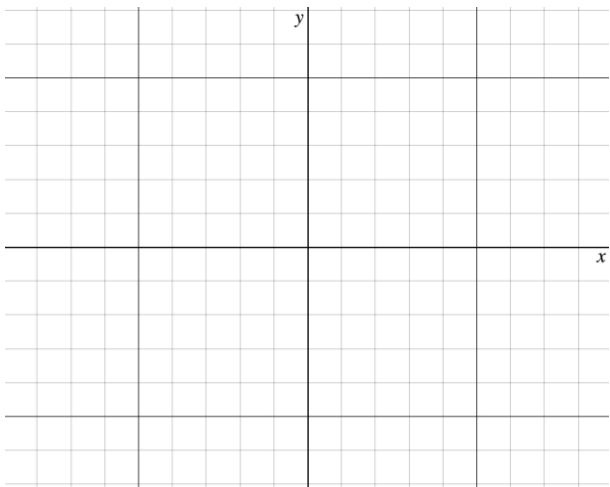
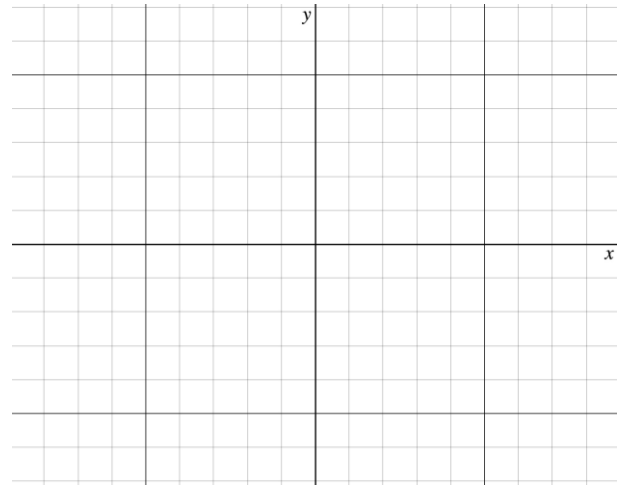
47. _____	64. _____	84. -----(See Attached Graph)-----
48. _____	65. _____	85. -----(See Attached Graph)-----
49. _____	66. _____	86. -----(See Attached Graph)-----
50. _____	67. _____	87. -----(See Attached Graph)-----
51a. _____	68. _____	88. -----(See Attached Graph)-----
51b. _____	69. _____	89. -----(See Attached Graph)-----
52a. _____	70. _____	90. -----(See Attached Graph)-----
52b. _____	71. _____	91a. _____
52c. _____	72. _____	91b. _____
53. _____	73. _____	91c. _____
54. _____	74. _____	91d. _____
55. _____	75. _____	92. _____
56. _____	76. _____	93. -----(See Attached Graphs)-----
57. _____	77. _____	94. -----(See Attached Graphs)-----
58. _____	78. _____	95a. _____

59. _____	79. _____	95b. _____
60. _____	80. _____	95c. _____
61. _____	81. _____	96a. _____
62. _____	82. _____	96b. _____
63. _____	83. -----(See Attached Graph)-----	96c. _____

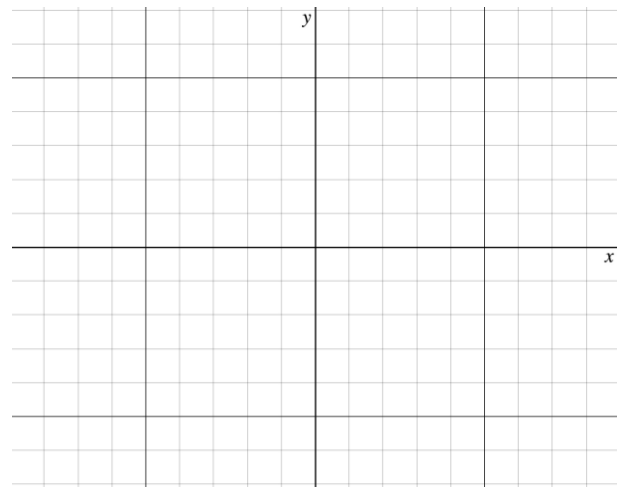
97. _____	99a. _____	100a. _____
98a. _____	99b. _____	100b. _____
98b. _____	99c. _____	100c. _____
98c. _____	99d. _____	100d. _____
98d. _____	99e. _____	100e. _____



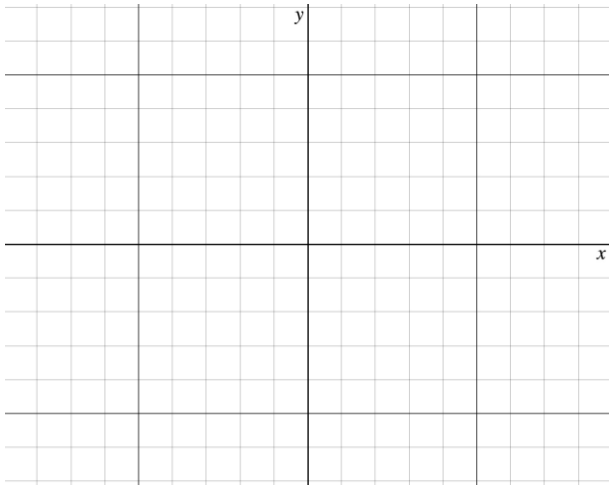
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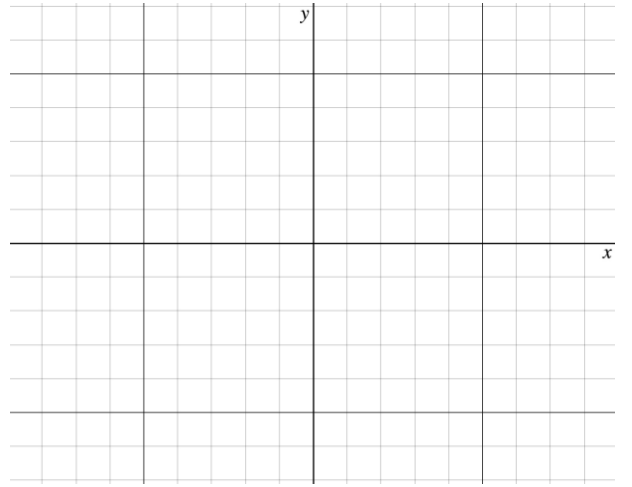
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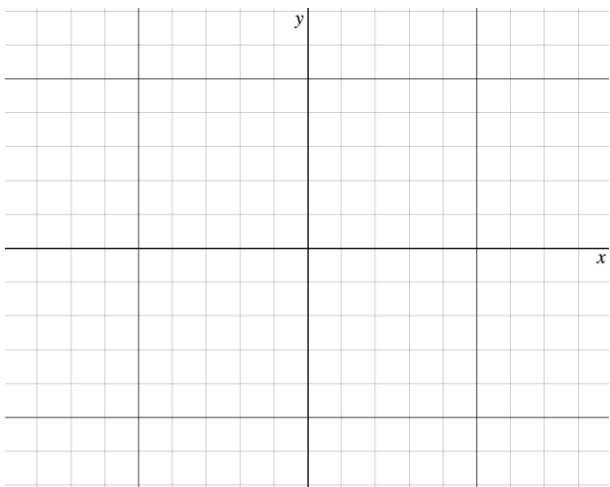
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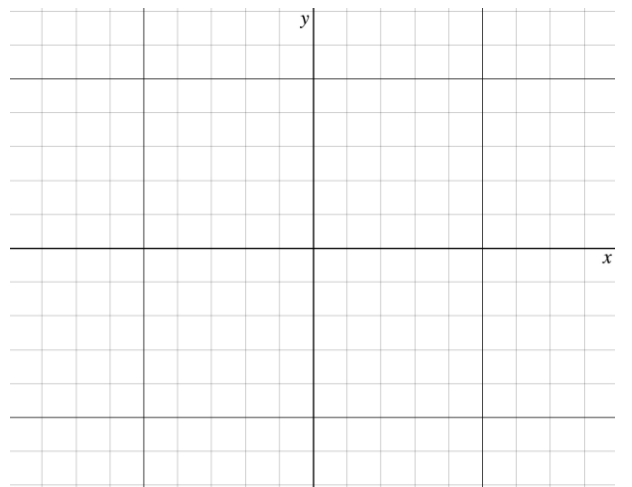
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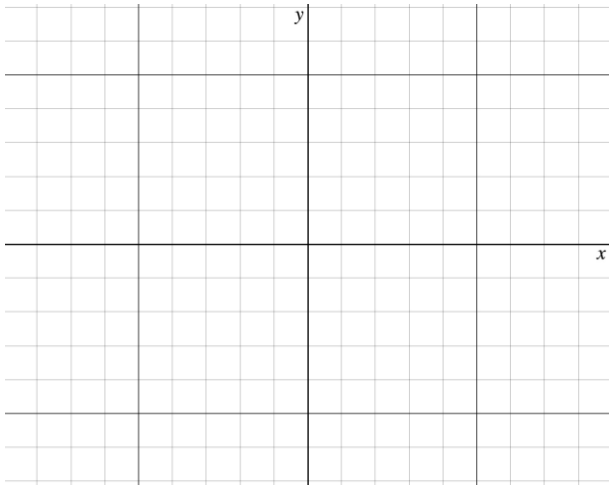
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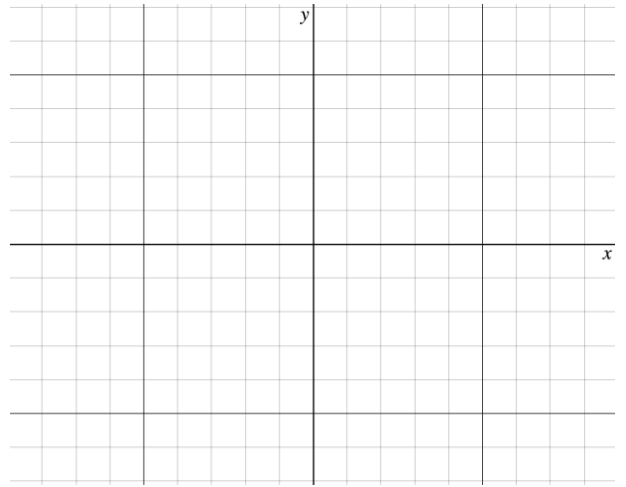
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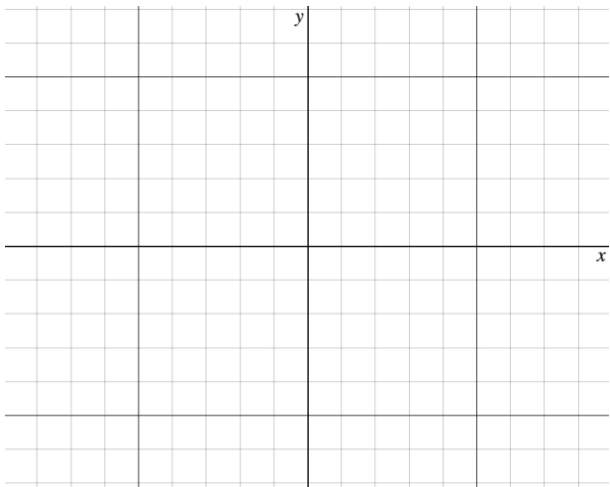
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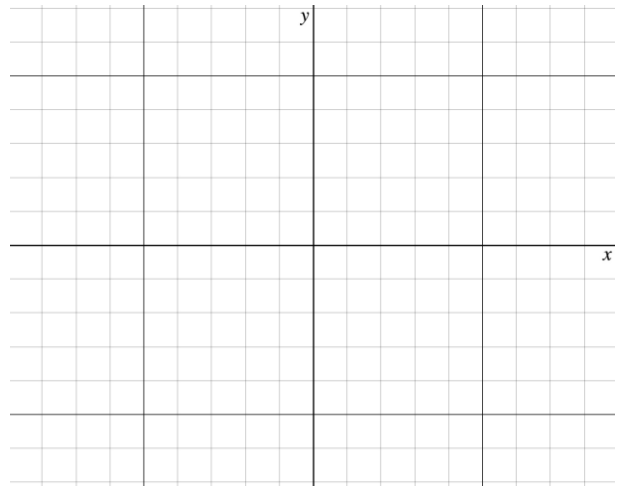
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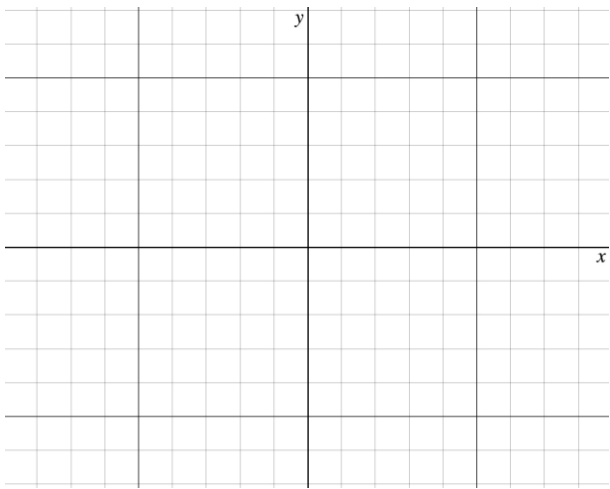


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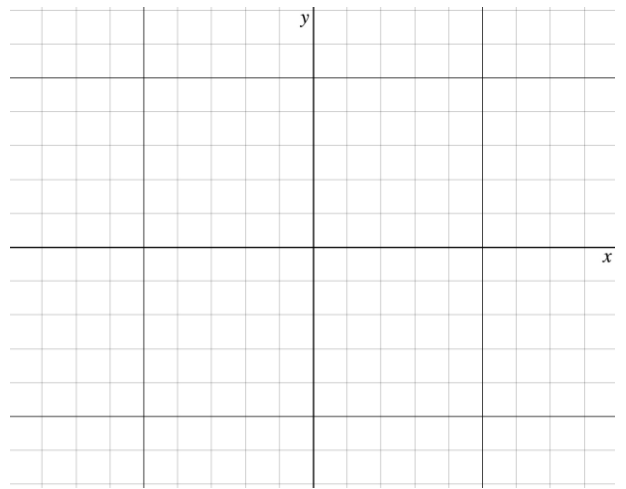


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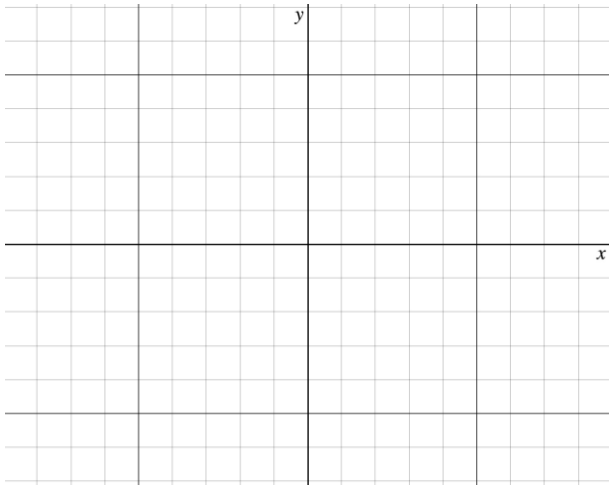




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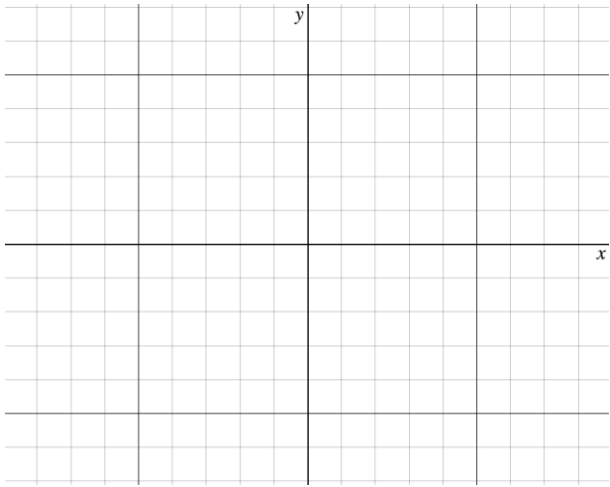
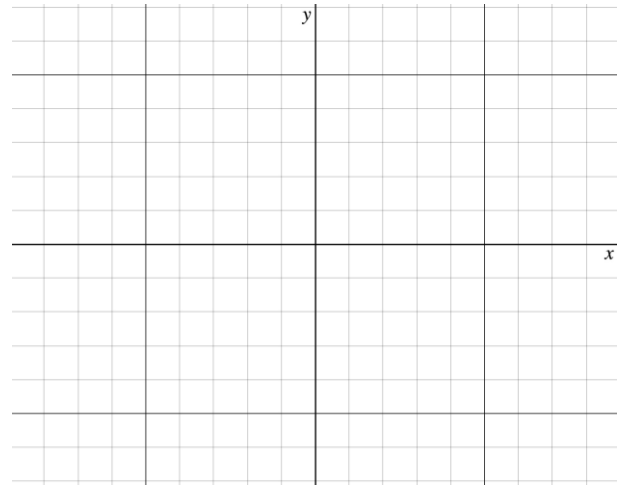


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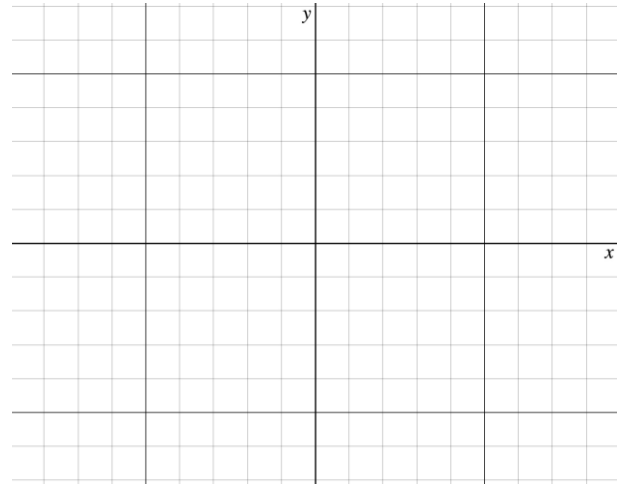


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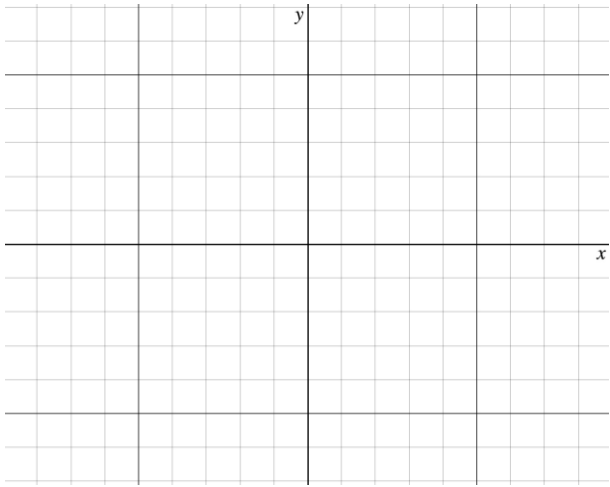
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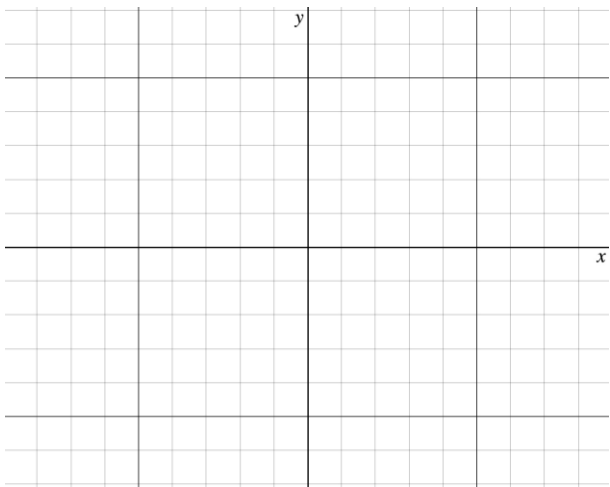
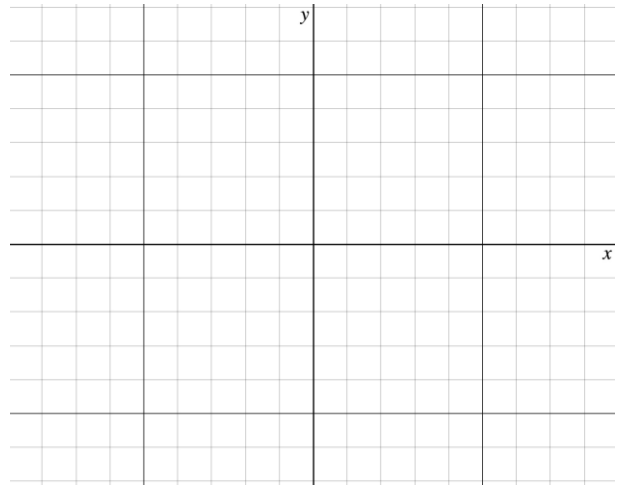


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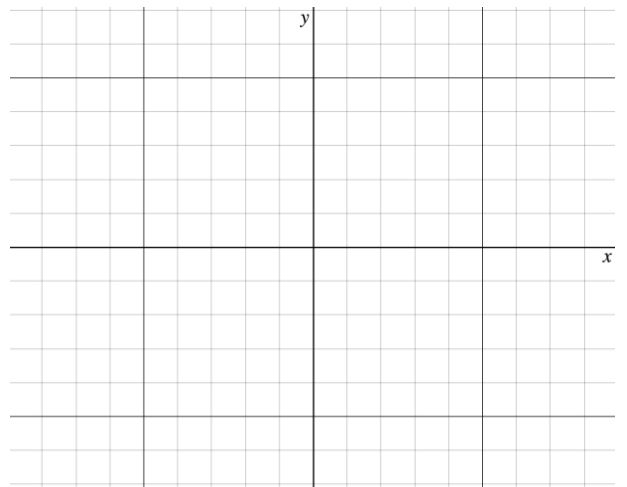


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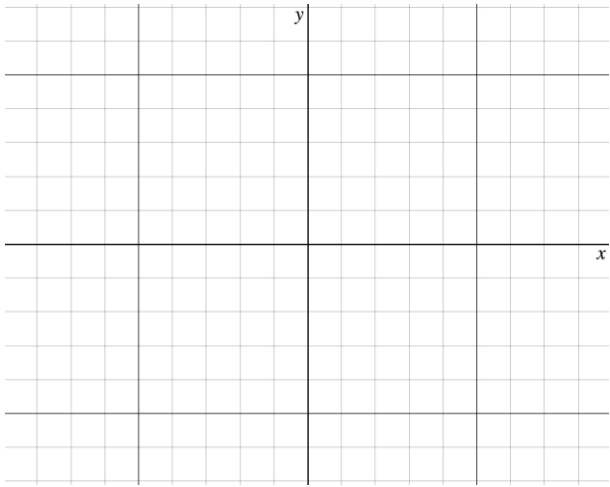
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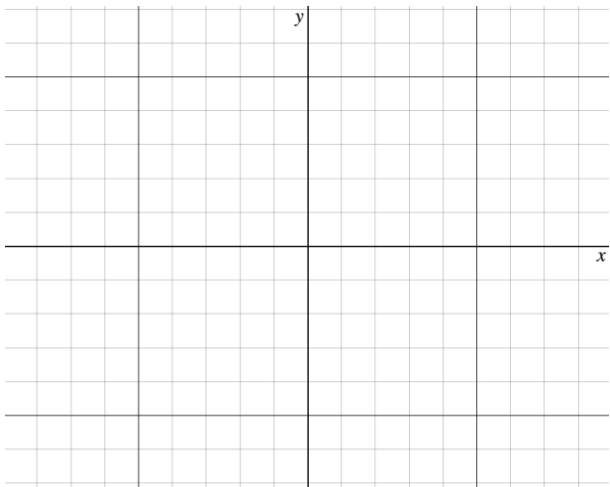
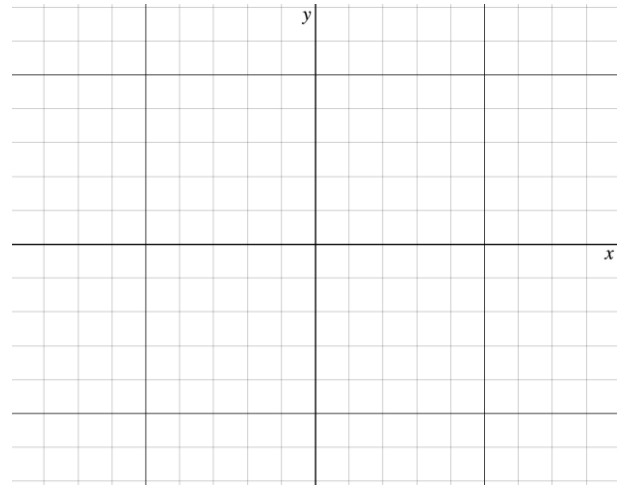
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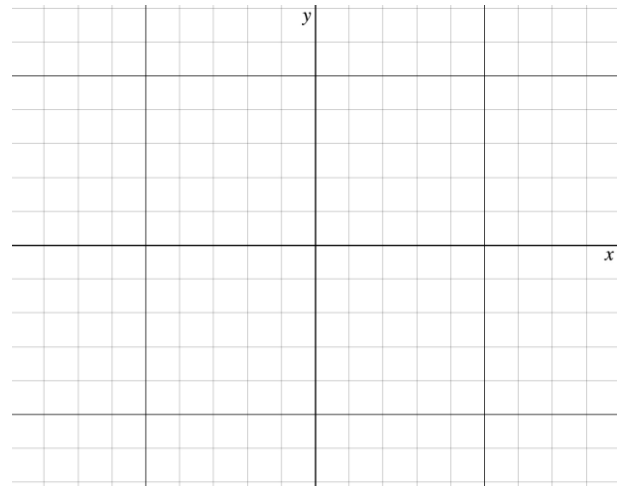
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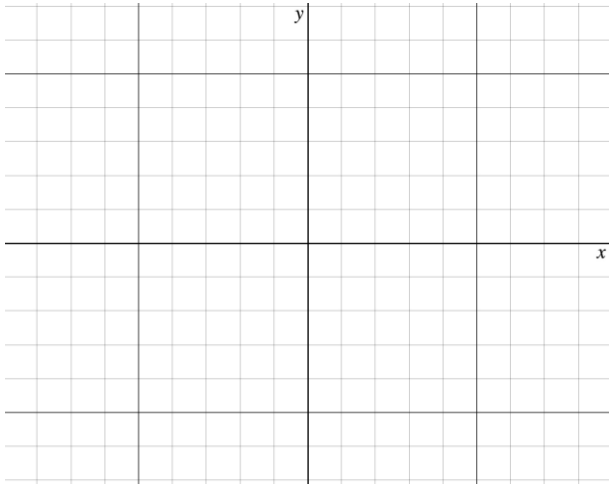
85. 86.



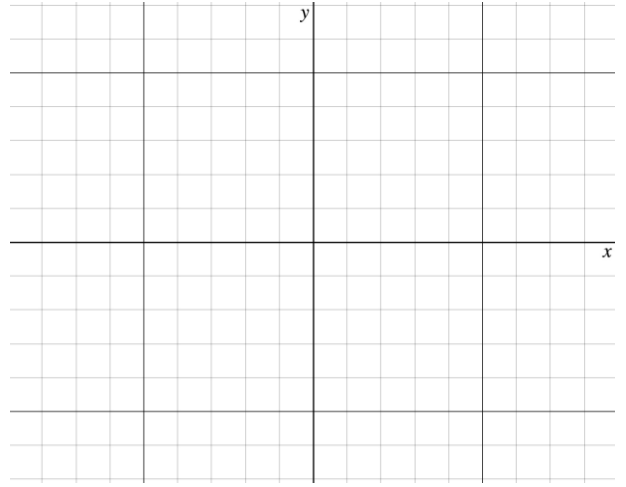
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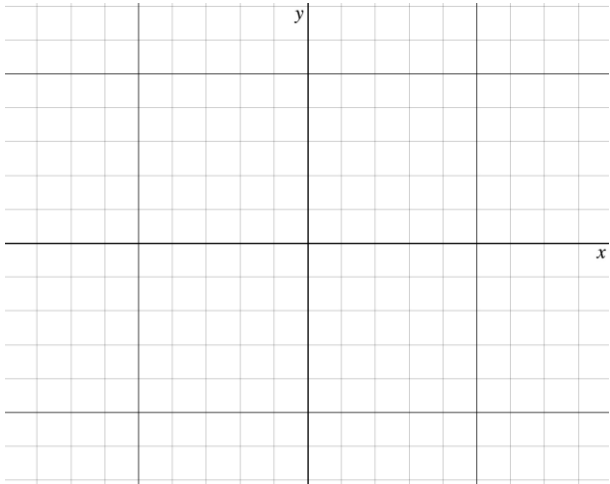
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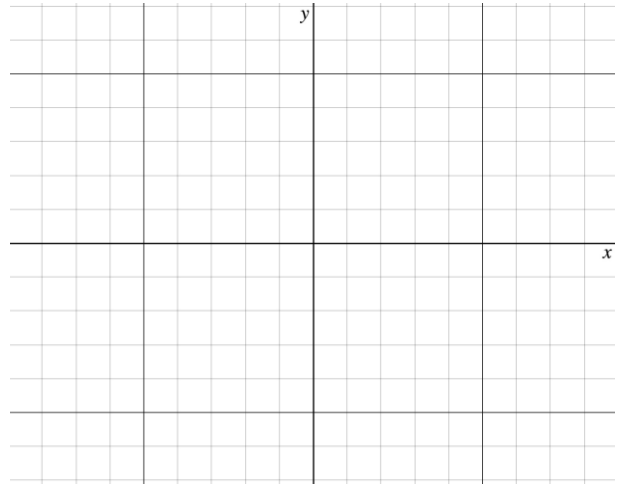
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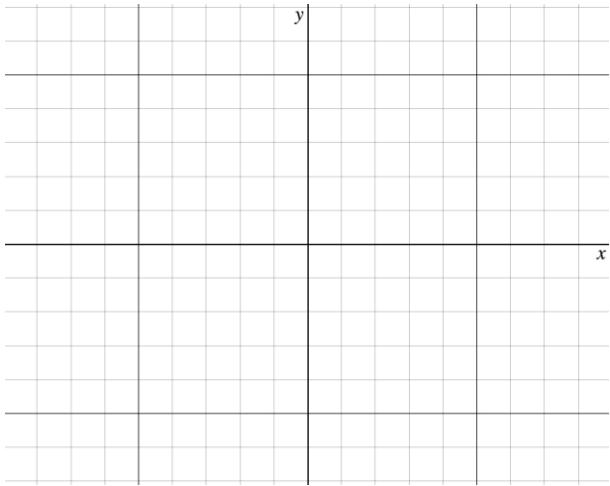
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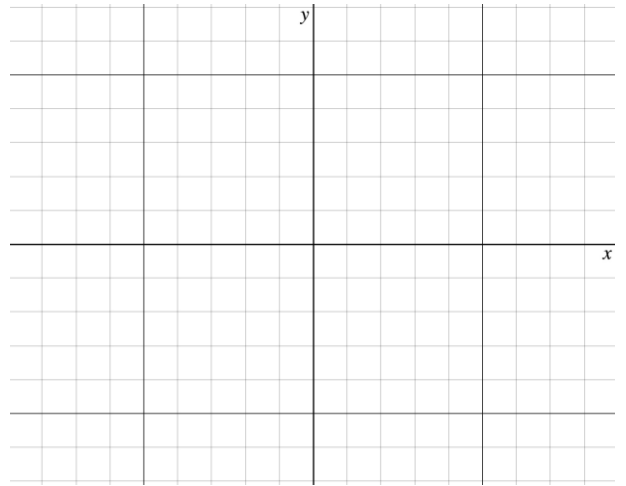
93.  
(a)



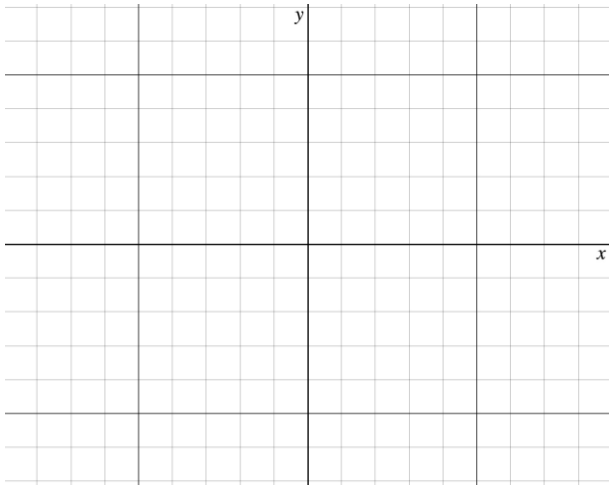
93.



93.  
(b)



93.  
(c)

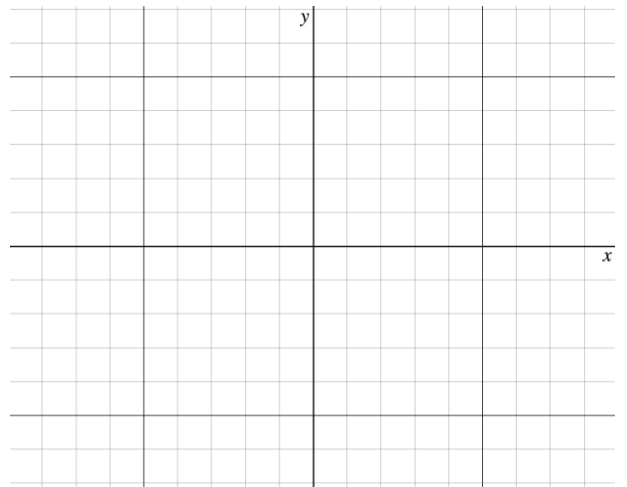
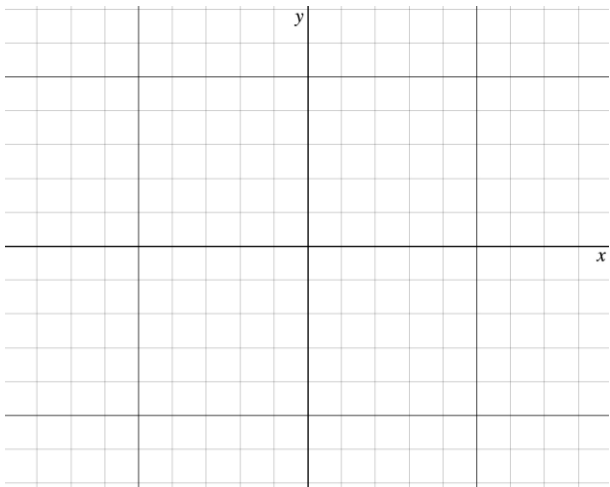
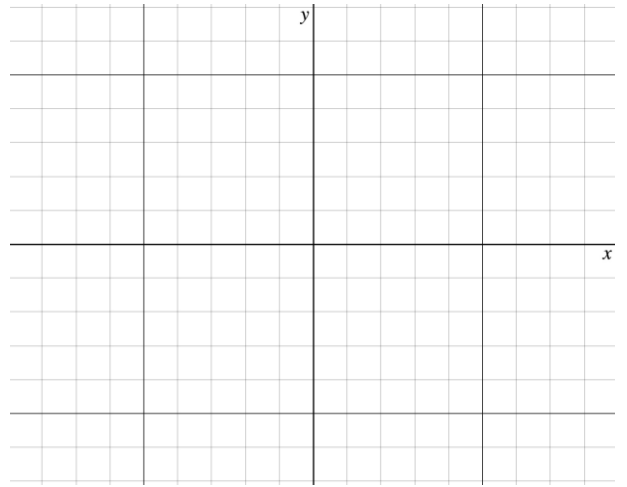


93.

(d)

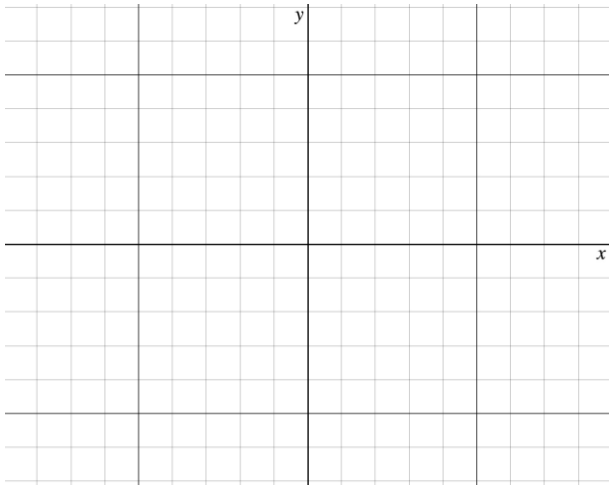
93.

(e)



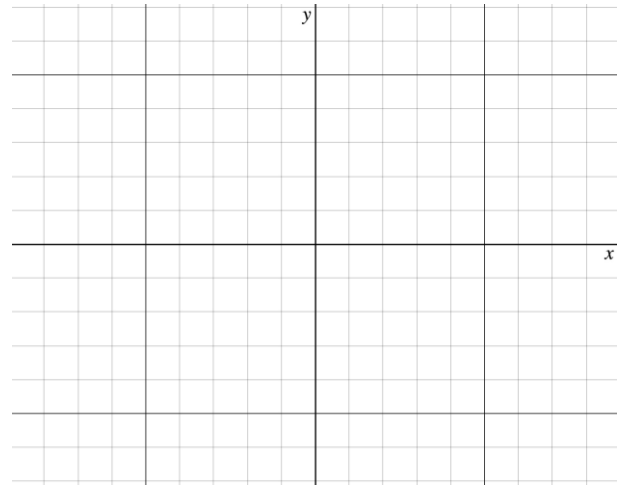
93.  
(f) (a)

94.



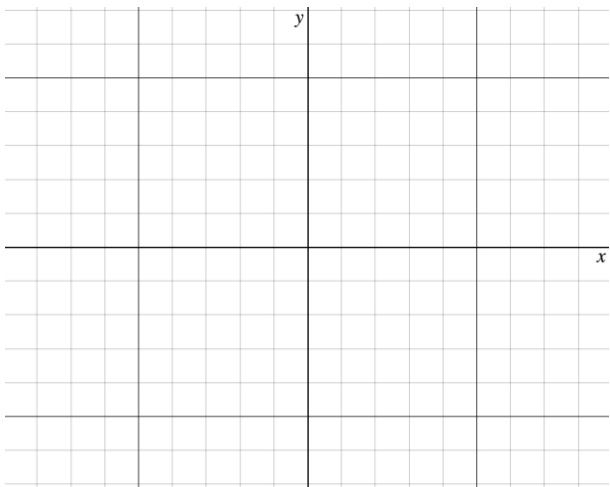
94.

(b)



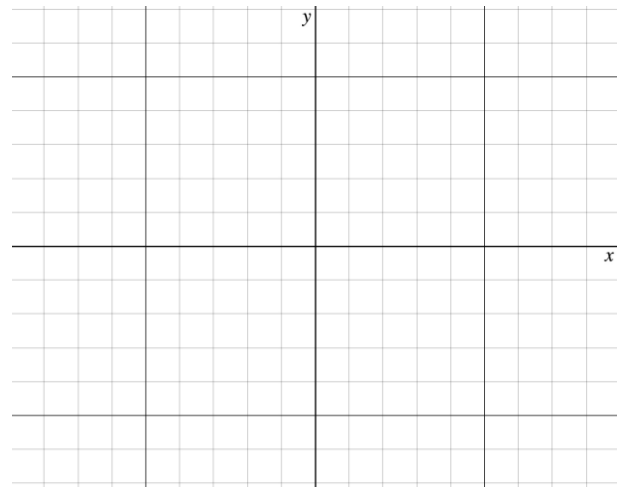
94.

(c)



94.

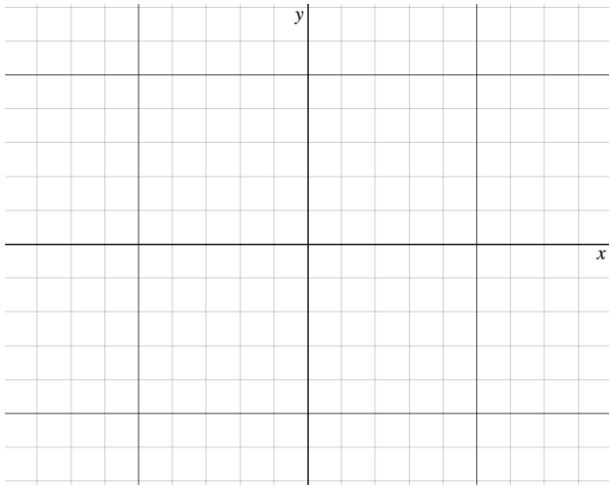
(d)



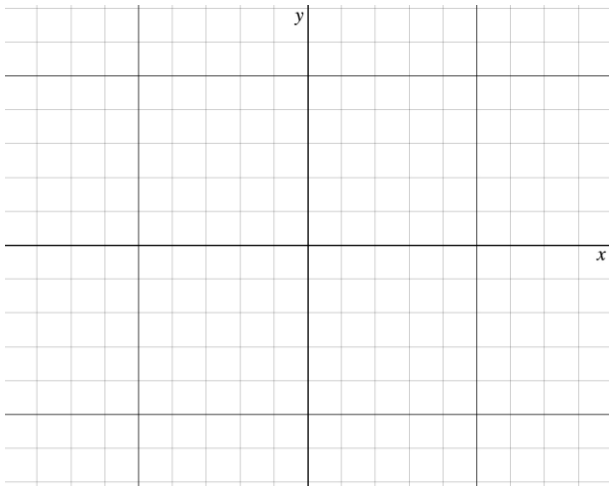
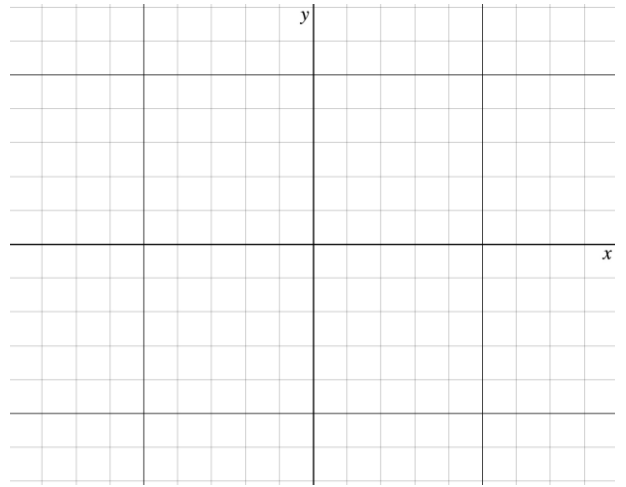
94.

(e)

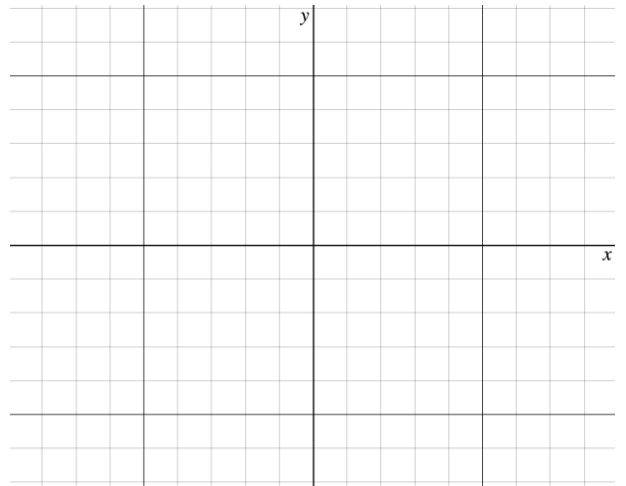




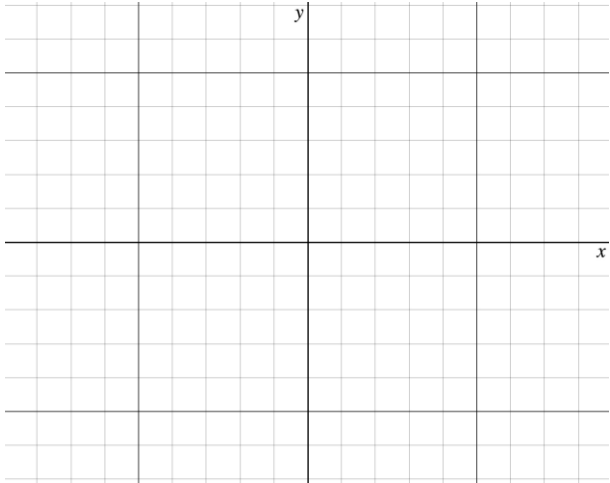
94. 94.  
(f) (g)



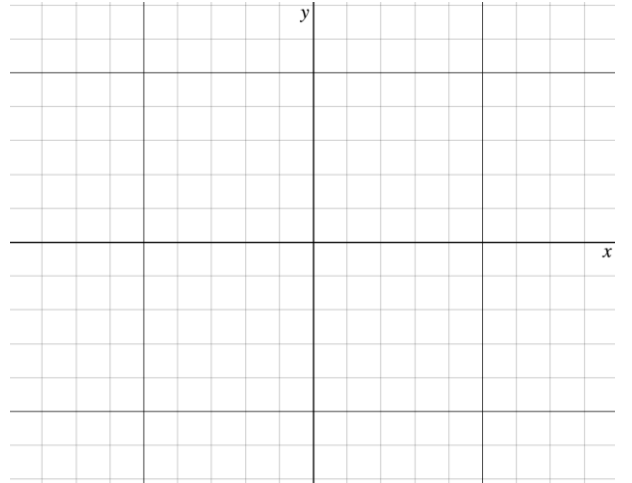
95.



96.



97.



98.